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# Transonic Panel Method to Determine Loads on Oscillating Airfoils with Shocks

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**A two-dimensional method is described that combines supersonic and subsonic linear lifting surface theories, both based on the velocity potential panel approach, while at the same time the effect of the moving shock is taken into account. For an NACA 64A006 airfoil transonic results are presented in comparison with results of an improved version of the LTRAN2 code, developed at NLR. The good agreement obtained shows the feasibility of the method, while the low computer cost makes this approach very attractive.**

## Nomenclature

| | |
|---|---|
| $c$ | = airfoil chord |
| $C_p$ | = pressure coefficient |
| $C_p^*$ | = critical pressure coefficient |
| $h$ | = unsteady motion of airfoil, $y=h(x,t)$ |
| $k$ | = reduced frequency, $k=\omega\ell/U_\infty$ |
| $k_{b,c}$ | = unsteady lift coefficient for pitching airfoil $b$ or airfoil with oscillating flap $c$, Eq. (23) |
| $\ell$ | = semichord, $\ell=c/2$ |
| $m_\ell$ | = source strength on $\ell$th panel |
| $m_{b,c}$ | = unsteady moment coefficient for pitching airfoil $b$ or airfoil with oscillating flap $c$, Eq. (24) |
| $M$ | = local Mach number, Eq. (2) |
| $M_\infty$ | = freestream Mach number |
| $M_1$ | = supersonic Mach number |
| $M_2$ | = subsonic Mach number |
| $n_{b,c}$ | = unsteady hinge moment coefficient for pitching airfoil $b$ or airfoil with oscillating flap $c$, Eq. (25) |
| $T$ | = time, nondimensionalized with $\ell/U_\infty$ |
| $U_\infty$ | = freestream velocity |
| $U_s$ | = shock velocity, Eq. (7) |
| $x,y$ | = orthogonal coordinates, nondimensionalized with $\ell$ |
| $x_m$ | = pitch axis |
| $x_r$ | = hinge axis |
| $X_s$ | = shock position |
| $\alpha$ | = angle of attack |
| $\alpha_s$ | = shock angle, Eq. (7) |
| $\gamma$ | = ratio of specific heat |
| $\lambda_1$ | = unsteady shock displacement, Eq. (21) |
| $\Phi$ | = disturbance velocity potential, nondimensionalized with $U_\infty\ell$ |
| $\omega$ | = angular frequency |
| $\Delta$ | = jump operator across a slit at $y=0$ |
| $[\![\ ]\!]$ | = jump operator across a slit at the shock position |
| $\langle\ \rangle$ | = average operator across a slit at the shock position |

*Subscripts*

| | |
|---|---|
| 0 | = mean steady-state value |
| 1 | = first harmonic component |
| + | = value at $y=0+$ |
| − | = value at $y=0-$ |

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## I. Introduction

RECENTLY, several methods have been developed to calculate the aerodynamic loads on oscillating wings in transonic flow. As yet, most of them are limited to applications to oscillating airfoils, while they also suffer from unattractively long computation times. These drawbacks have impelled NLR's activities to search for methods that are more suited to application in aeroelastic calculations.

A classification of existing methods is possible by a distinction between methods based on a linear-field equation governing the unsteady flow and methods using a nonlinear-field equation. Several finite-difference methods belonging to the last category have been developed successfully. A typical example is the LTRAN2 code for two-dimensional flow.[1,2] However, as stated before, such methods are rather costly as far as computer time is concerned, and therefore less practical in routine aeroelastic calculations.

The methods based on a linear-field equation, employ the separation of the flowfield into a mean steady and a small unsteady part where the mean steady flowfield determines the coefficients of the linear equation. A further distinction can be made into methods that try to model the coefficients of the linear-field equation as well as possible, resulting in finite-difference techniques[3,4] or in a field-integral equation[5]; and the methods that model the coefficients in a rather coarse way, thus permitting the use of slightly modified linear lifting surface theories.[6-8] The former methods still have the disadvantage of relatively high computer cost and at the moment they are not well developed. For this reason at the NLR it was decided to start the development of an unsteady transonic method based on the second approach.

The present method combines supersonic and subsonic linear lifting surface theories (zero wing thickness) in such a way that the effect of the moving shock is properly accounted for. A basic difference between the present approach and the one of Ref. 8 is the adoption of velocity potential panel methods as the building stones of the method, which permits application to both two- and three-dimensional wing configurations.

The present paper reports on the first phase of this development: a method for thin symmetric airfoils, oscillating in transonic flow of which the steady part is divided into a supersonic constant Mach number region and a subsonic constant Mach number region separated by a straight shock. An analogous model has been proposed earlier by Eckhaus[9] and recently applied successfully by Williams.[8] A derivation of the shock relation is given that contributes to the understanding of the shock load, caused by the shock motion in linearized methods.

A comparison is made with results of a version of the LTRAN2 code developed at NLR. The good agreement

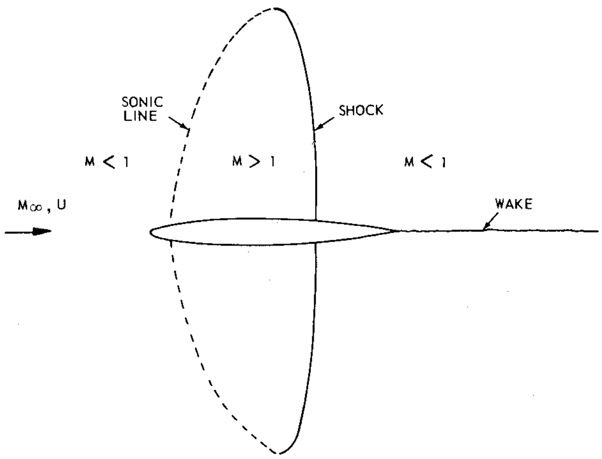


**Fig. 1 Transonic flow about thin symmetric airfoil.**

obtained shows the feasibility of the method while the low computer cost makes this approach very attractive.

## II. General Aerodynamic Analysis

The transonic flow is considered around an oscillating thin airfoil with a well-developed shock wave in an oncoming flow $U_\infty$ (Fig. 1). To describe the flowfield, a coordinate system $(x,y)$ is chosen fixed to the mean position of the airfoil with the $x$ axis in streamwise direction. Assuming isentropic and irrotational nonviscous flow a disturbance velocity potential $\Phi$ can be defined that has to satisfy the nonlinear-field equation governing the unsteady flowfield induced by a thin airfoil in transonic flow[10]

$$(1-M^2)\Phi_{xx}+\Phi_{yy}-2M_\infty^2\Phi_{xT}-M_\infty^2\Phi_{TT}=0 \tag{1}$$

where $M_\infty$ is the freestream Mach number and $M$ the local Mach number

$$M^2=M_\infty^2+(\gamma+1)M_\infty^2\Phi_x \tag{2}$$

The condition for tangential flow at the upper and lower surfaces of the airfoil is expressed by

$$\Phi_y(x,y=\pm 0,T)=\alpha_\pm(x,T)+\frac{\partial h(x,T)}{\partial T} \tag{3}$$

where $h$ denotes the normal displacement of the mean surface. The pressure is given by the linearized Bernoulli equation

$$C_p=-2\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial T}\right)\Phi \tag{4}$$

The pressure jump between both sides of the wake is zero

$$\Delta C_p(x)=C_p(x,y=0+)-C_p(x,y=0-)=0 \quad x\geq 1 \tag{5}$$

For a moving shock located at $x=X_s(y,T)$ the consistent shock relation to Eq. (1) is[11]

$$[\![\Phi]\!]=0 \quad -\langle 1-M^2\rangle=\alpha_s^2+M_\infty^2(2U_s-U_s^2) \tag{6}$$

where $\langle A\rangle$ denotes the average and $[\![A]\!]$ the difference of any quantity $A$ across the shock. $\alpha_s$ is the shock angle and $U_s$ the shock velocity

$$\alpha_s=\frac{\partial X_s}{\partial y}, \quad U_s=\frac{\partial X_s}{\partial T} \tag{7}$$

Further it is required that the disturbances propagate outward from the airfoil at infinity (Sommerfeld condition) and that at the trailing edge the Kutta condition is imposed. The aforementioned variables are all made dimensionless with the semichord length $\ell$ and the oncoming flow velocity $U_\infty$. The field Eq. (1) and the mentioned boundary conditions are the boundary-value problem in the aforementioned nonlinear approach.

Next the assumption is made that the airfoil motion is a small amplitude harmonic oscillation about a mean position

$$h=h_1(x)e^{ikT} \qquad \alpha_\pm=\alpha_{0_\pm}+\frac{dh_1}{dx}e^{ikT} \tag{8}$$

where $h_1$ denotes the amplitude of the oscillation and $k$ the reduced frequency, $k=\omega\ell/U_\infty$.

Then the flowfield is periodic with period $2\pi/k$ and a Fourier series expansion is possible

$$\Phi(x,y,T)=\Phi_0(x,y)+\Phi_1(x,y)e^{ikT}+\ldots \tag{9}$$

A straight-forward linearization by assuming $\ldots\ll\Phi_2\ll\Phi_1\ll\Phi_0$ is not possible since this assumption is violated in the neighborhood of the moving shock. However, as this region is supposed to be small, the second and higher harmonic components in the total airloads are small compared with the first harmonic components. Therefore, the present analysis is restricted to $\Phi_0$ and $\Phi_1$ where the subscripts 0 and 1 denote mean and first harmonic potentials. The field equations corresponding to the two first terms of expression Eq. (9) are

$$(1-M_0^2)\Phi_{0_{xx}}+\Phi_{0_{yy}}=0 \tag{10a}$$

$$[(1-M_0^2)\Phi_{1_x}]_x+\Phi_{1_{yy}}-2ikM_\infty^2\Phi_{1_x}+k^2M_\infty^2\Phi_1=0 \tag{10b}$$

where

$$M_0^2=M_\infty^2+(\gamma+1)M_\infty^2\Phi_{0_x} \tag{11}$$

The conditions for tangential flow become

$$\Phi_{0_y}(x,y=\pm 0)=\alpha_{0_\pm} \tag{12a}$$

$$\Phi_{1_y}(x,y=\pm 0)=\frac{\partial h_1}{\partial x}+ikh_1 \tag{12b}$$

and the pressure coefficients are given by the expressions

$$C_{p_0}=-2\Phi_{0_x} \tag{13a}$$

$$C_{p_1}=-2(\Phi_{1_x}+ik\Phi_1) \tag{13b}$$

The conditions in the wake are

$$\Delta C_{p_0}=0 \tag{14a}$$

$$\Delta C_{p_1}=0 \tag{14b}$$

So far Eqs. (10-14) have been derived straight from Eqs. (1-5). To set up consistent shock relations from Eq. (6), however, requires a more subtle reasoning. Equation (6) has to be satisfied at the periodically moving shock of which the instantaneous position is

$$X_s=X_{s_0}+\lambda_1 e^{ikT}+\ldots \tag{15}$$

The flow is divided into an inner region, that is bounded by the extreme positions of the shock, and an outer region (see Fig. 2). It is possible to derive a shock relation for $\Phi_0$ and $\Phi_1$ that relates the values of $\Phi_0$ and $\Phi_1$ at the extreme shock positions by integrating Eq. (10) and the irrotationality

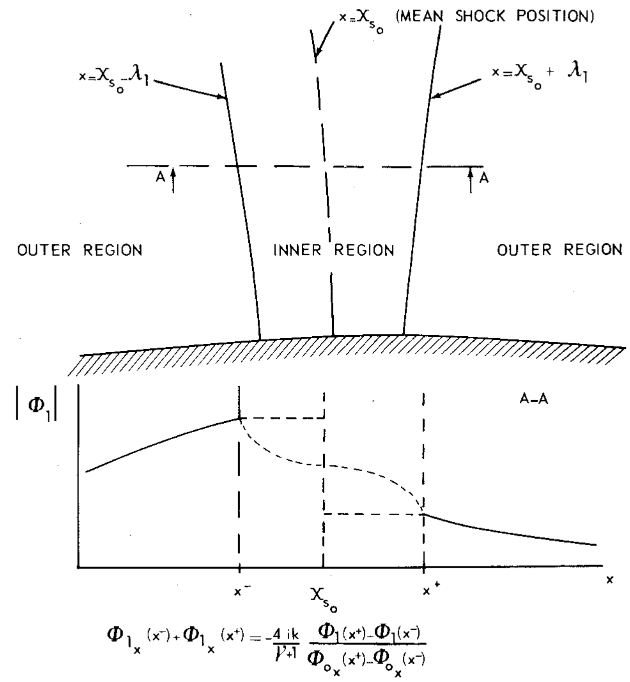


**Fig. 2 The shock relation.**

conditions

$$\Phi_{0_{xy}} = \Phi_{0_{yx}} \tag{16a}$$

$$\Phi_{l_{xy}} = \Phi_{l_{yx}} \tag{16b}$$

across the inner region according to Green's theorem. Then it follows, neglecting terms of order $(\lambda_l \Phi_0)$

$$\langle 1 - M_0^2 \rangle = -\alpha_{s_0}^2 \tag{17}$$

and neglecting terms of order $(\lambda_l \Phi_l)$

$$\langle \Phi_{l_x} \rangle = -\frac{2ik}{\gamma+1} \frac{[\![ \Phi_l ]\!]}{[\![ \Phi_{0_x} ]\!]} \tag{18}$$

where $\langle A \rangle$ and $[\![ A ]\!]$ now denote the average and the difference of any quantity $A$ in the outer region at the extreme shock positions. Equations (17) and (18) may consistently be applied at the mean (steady-state) shock position without violating the order of approximation, holding in mind, however, that they remain relations for only the flow in the outer region. Since all equations for $\Phi_0$ formerly indicated agree with the equations for the steady-state potential, $\Phi_0$ in the outer region may be chosen equal to the steady-state potential. Equation (18) is recognized as the Landahl[10] shock relation.

There is no need to consider the potential in the inner region as long as one is only interested in the unsteady airloads because the pressure can be integrated according to

$$\int_{x=X_{s_0}-\lambda_l}^{x=X_{s_0}+\lambda_l} C_{p_l} \mathrm{d}x = -2\int_{x=X_{s_0}-\lambda_l}^{x=X_{s_0}+\lambda_l} (\Phi_{l_x} + ik\Phi_l)\mathrm{d}x$$

$$= -2[\Phi_l(X_{s_0}+\lambda_l-0) - \Phi_l(X_{s_0}-\lambda_l+0)] + 0(k\lambda_l\Phi_l) \tag{19}$$

Since $\Phi_l$ is continuous through the boundaries of both regions, the shock load is finally

$$\int_{x=X_{s_0}-\lambda_l}^{x=X_{s_0}+\lambda_l} C_{p_l} \mathrm{d}x = -2[\![ \Phi_l ]\!] + 0(k\lambda_l\Phi_l) \tag{20}$$

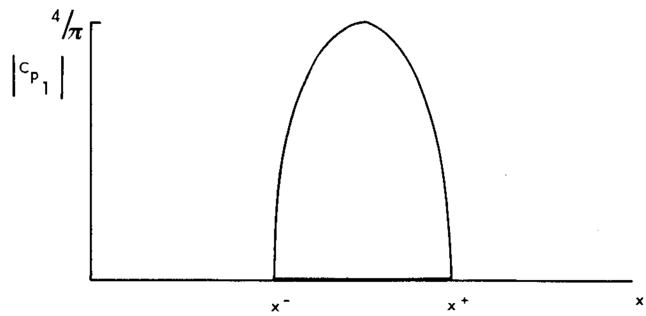


**Fig. 3 The first harmonic in the unsteady pressure due to the shock motion.**

where $[\![ \Phi_l ]\!]$ represents the difference of the potential $\Phi_l$ in the outer region at the extreme shock positions.

In summary, unsteady first harmonic airloads can be obtained after solving the potential $\Phi_l$ in the outer region which is a solution of the linear Eq. (10b) with boundary conditions Eqs. (12b) and (14b) and the Landahl shock relation Eq. (18). Unsteady first harmonic pressure distributions can be obtained outside the shock trajectory using the expression Eq. (13), but unfortunately not inside the shock trajectory while the problem there remains nonlinear ($\lambda_l$ and $\Phi_0$ are unknown).

A pressure distribution in the inner region can be derived if one assumes that a Taylor series expansion can be introduced for the steady-state potential. By matching the potential in the outer and inner region using the continuity of $\Phi_l$ at the extreme positions one can find the accompanying shock displacement

$$\lambda_l = -[\![ \Phi_l ]\!] / [\![ \Phi_{0_x} ]\!] \tag{21}$$

The unsteady pressure distribution in the inner region can be derived in the same way as in Ref. 12

$$C_{p_l} = \frac{4\lambda_l [\![ \Phi_{0_x} ]\!]}{\pi |\lambda_l|} \sqrt{1 - \frac{(x - X_{s_0})^2}{|\lambda_l|^2}} + 0(k\Phi_l) \tag{22}$$

The distribution of $C_p$ along the chord has been indicated in Fig. 3. It has the same shape as the peaks that can be recognized in experimental pressure distributions, because usually only the first harmonic component is measured.

## III. Unsteady Airloads

The first harmonic of the lift, moment, and flap hinge moment coefficients are obtained by integrating the pressure difference along the airfoil and put into standard AGARD notation[13]

$$k_{b,c} = -\frac{1}{2\pi}\int_{-1}^{1} \Delta C_{p_l} \mathrm{d}x = \frac{1}{\pi}\Delta\Phi_l(1) + \frac{ik}{\pi}\int_{-1}^{1} \Delta\Phi_l \mathrm{d}x \tag{23}$$

$$m_{b,c} = -\frac{1}{2\pi}\int_{-1}^{1} (x - x_m)\Delta C_{p_l} \mathrm{d}x = \frac{1}{\pi}(1 - x_m)\Delta\Phi_l(1)$$

$$-\frac{1}{\pi}\int_{-1}^{1} \Delta\Phi_l[1 - ik(x - x_m)]\mathrm{d}x \tag{24}$$

and

$$n_{b,c} = -\frac{1}{2\pi}\int_{x_r}^{1} (x - x_r)\Delta C_{p_l} \mathrm{d}x = \frac{1}{\pi}(1 - x_r)\Delta\Phi_l(1)$$

$$-\frac{1}{\pi}\int_{x_r}^{1} \Delta\Phi_l[1 - ik(x - x_r)]\mathrm{d}x \tag{25}$$

The contribution of the inner region (i.e., the shock load) is included in these expressions.

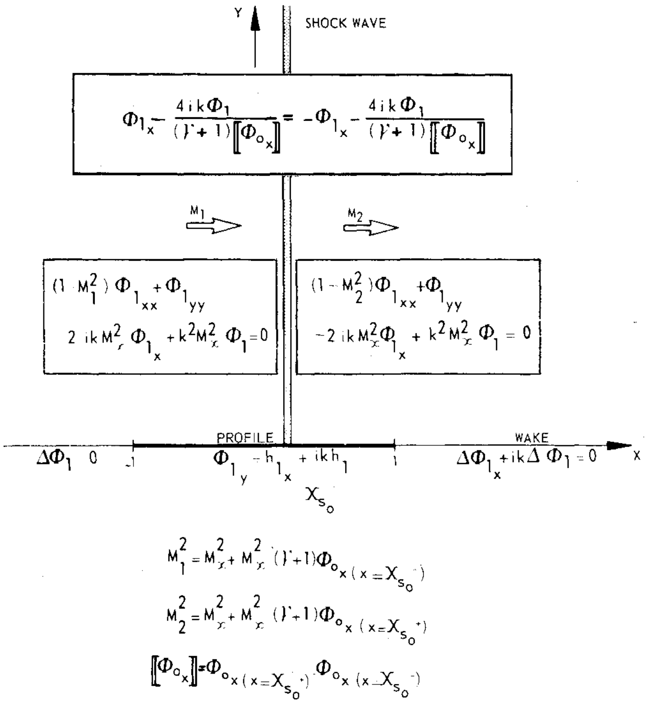


**Fig. 4 The boundary-value problem.**

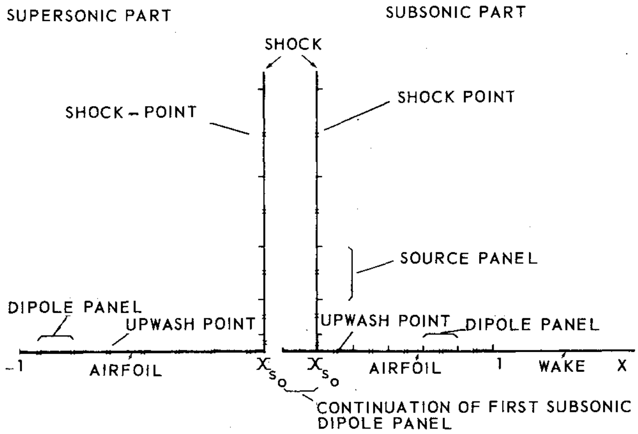


**Fig. 5 Model of wing and flow.**

## IV. Further Simplifications and Computational Method

The computational method is based on a simplified representation of a mean steady symmetric flowfield that is divided into a supersonic constant Mach number region and a subsonic constant Mach number region separated by a straight normal shock. This steady field is assumed to be known from theory or experiment. Of course, this simplification may limit the range of applicability of the method. On that subject some observations can be made.

1) For reduced frequencies $k \gg 0(\Phi_0)$ the effects of the nonhomogeneity of the steady flow become less important, and linear lifting surface theories (zero wing thickness) will suffice.

2) When $k = 0(\Phi_0)$, all the terms in Eq. (10b) (except $k^2 M_\infty^2 \Phi_I$) are of the same order of magnitude in the transonic small perturbation limit, so a proper description of the steady flowfield is required.

It is expected that, in spite of the crudeness of the present assumption concerning the Mach number distribution, the computational model constitutes an improvement over classical linear theory, but verification is needed with results for more exact theory. One might improve the range of applicability by using "local linearization" concepts.[14]

The unsteady flowfields on both sides of the shock are described by linearized equations with constant coefficients that depend on the steady field (see Fig. 4). The coupling between the two regions is realized by the shock relation Eq. (18) applied at the mean position of the shock.

As the shock can be assumed to be impenetrable for disturbances from the subsonic region, the description of the unsteady supersonic flowfield may be treated as a separate boundary-value problem. A solution is found by specifying on the chord line of the airfoil a panel distribution, with each panel containing a constant, yet unknown, dipole distribution representing the velocity potential jump between upper and lower surface. The strength of the dipole distribution is found by specifying a tangential flow condition compatible with the motion of the airfoil. This results in the algebraic system

$$\sum_i DU_{ij} \Delta\Phi_{I_i} = \left[\frac{\partial h_I}{\partial x} + ikh_I\right]_{x=x_j} \tag{26}$$

where $DU_{ij}$ are influence coefficients representing the contribution of the dipole distribution on panel $i$ to the upwash at panel $j$. $x_{ij}$ denotes the upwash points on the airfoil.

With the dipole distribution known, the flow conditions at the upstream side of the shock can be determined and substituted in the shock relation Eq. (18)

$$\left[\Phi_{I_x} + \frac{4ik\Phi_I}{(\gamma+1)\,[\![\Phi_{0x}]\!]}\right]_{\substack{x=X_{s0}\\ y=y_\ell}} = \sum_i S_{i\ell}\Delta\Phi_{Ii} \tag{27}$$

where $S_{i\ell}$ are influence coefficients representing the contribution to the variables in the shock relation upstream of the shock and $(X_{s0}, y_\ell)$ denotes the points where the shock relation is applied.

Downstream of the shock the boundary-value problem is solved by assuming again a distribution of constant dipole panels on the chord line and in addition a set of constant source panels on the shock. The boundary conditions at the downstream side of the shock follow from the compatibility condition across the shock and on the chord line from the airfoil motion. For the wake a zero pressure jump is prescribed. Now the algebraic system becomes

$$\sum_i DD_{ij}\Delta\Phi_{Ii} + \sum_m SD_{mi}m_m + WD_j\Delta\Phi_I(x=1)$$

$$= \left[\frac{\partial h_I}{\partial x} + ikh_I\right]_{x=x_j} \sum_i DS_{i\ell}\Delta\Phi_{Ii} + \sum_m SS_{m\ell}m_m$$

$$+ WS_\ell\Delta\Phi_I(x=1) = \sum_i S_{i\ell}\Delta\Phi_{Ii} \tag{28}$$

where $DD_{ij}$, $SD_{mi}$, $WD_j$, $DS_{i\ell}$, $SS_{m\ell}$, and $WS_\ell$ are influence coefficients. $DD_{ij}$, $SD_{mi}$, and $WD_j$ represent the contribution to the upwash of the dipole panels, the source panels, and the wake, respectively. $m_m$ denotes the source strength. $DS_{i\ell}$, $SS_{m\ell}$, and $WS_\ell$ represent the contribution to the variables in the shock relation downstream of the shock of the dipole panels, the source panels, and the wake, respectively. $\Delta\Phi_I(x=1)$ denotes the potential jump at the trailing edge. The algebraic system Eq. (28) is completed with the explicit requirement of zero pressure jump at the trailing edge (Kutta condition) that determines $\Delta\Phi_I(x=1)$.

After the solution of this second algebraic system the potential jump on the whole airfoil is known and total unsteady loading coefficients can be obtained using Eqs. (23-25).

Additional comments on this method are as follows:

1) The influence coefficients can easily be derived from linear lifting surface theory.[10]

2) Because of symmetry the computational method has been reduced to the upper half plane.

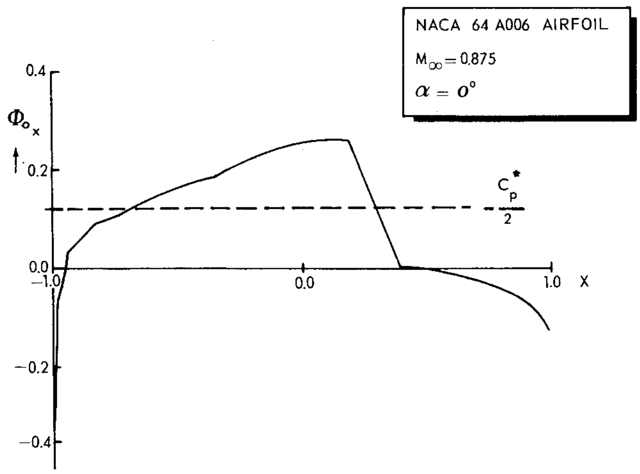


**Fig. 6 Steady flowfield.**

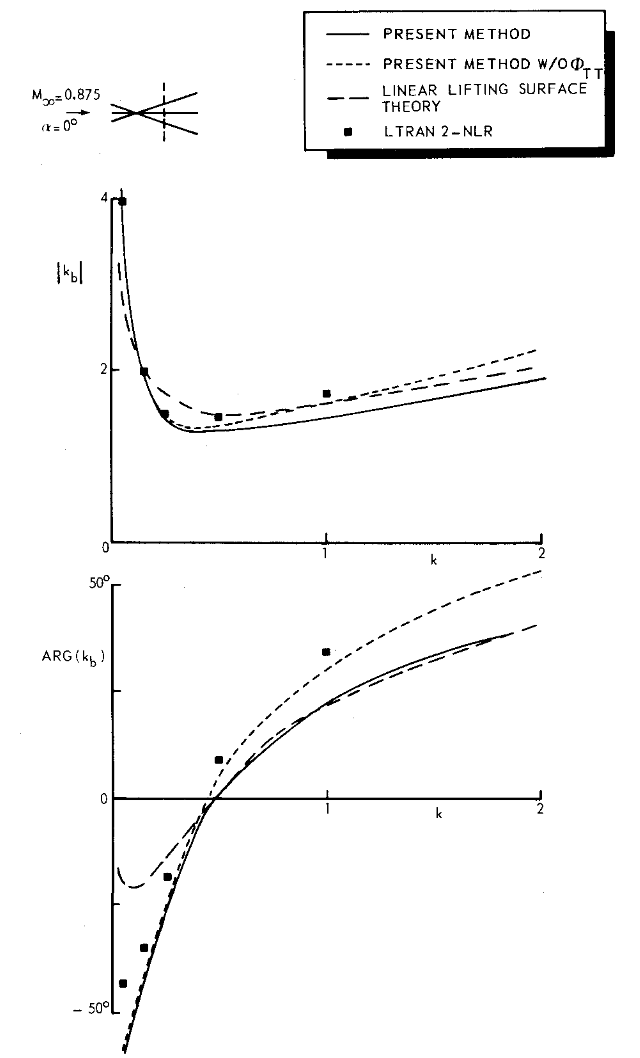


**Fig. 7 Unsteady lift coefficient on a pitching NACA 64A006 airfoil.**

3) At the shock foot the subsonic potential jump across the airfoil is continued one panel size into the supersonic region (Fig. 5) in order to avoid the solution becoming unacceptably sensitive to the size of the source and dipole panels. The usefulness of this extended panel has been shown by numerical experiments.

4) To avoid an unnecessarily large number of source panels on the shock the panel size is increased with the vertical distance.

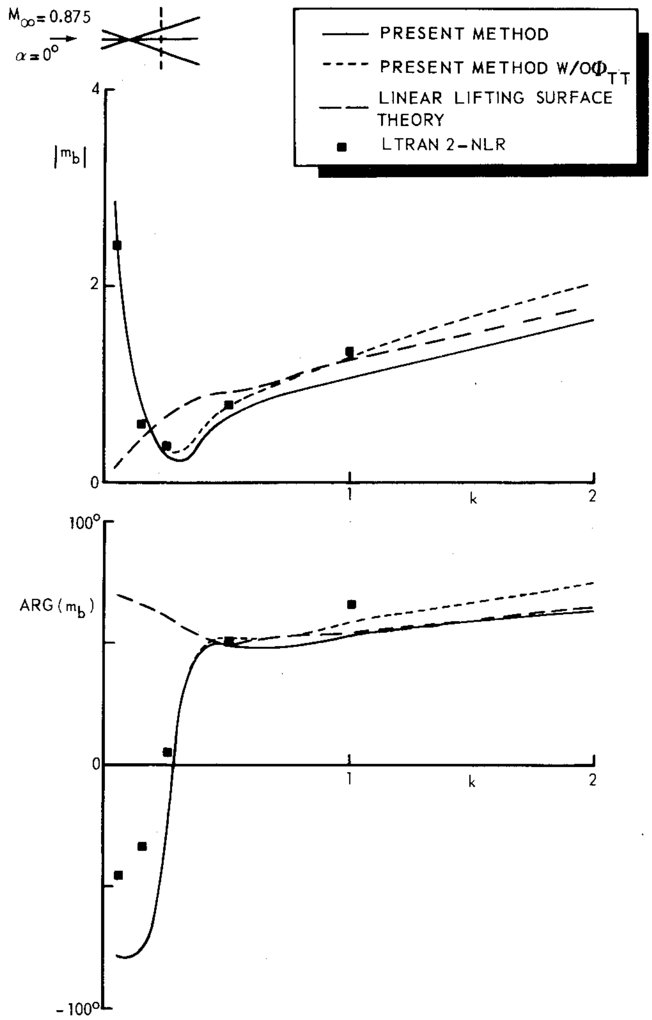


**Fig. 8 Unsteady moment coefficient on a pitching NACA 64A006 airfoil.**

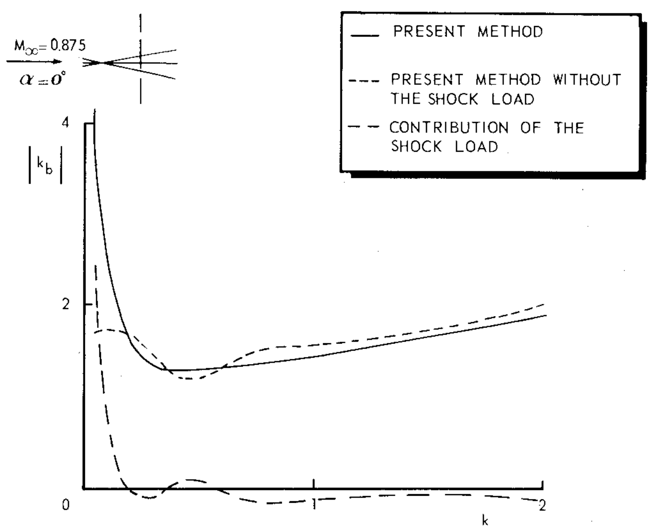


**Fig. 9 Unsteady lift coefficient on a pitching NACA 64A006 airfoil showing the contribution of the shock load.**

## V. Calculations

Unsteady loads calculations have been performed for an NACA 64A006 airfoil in a flow with $M_\infty = 0.875$ and $\alpha_0 = 0$ deg. The vibration modes are pitching about the quarter-chord point and flap rotation about the flap leading edge at the three-quarter-chord point. To assess the applicability of

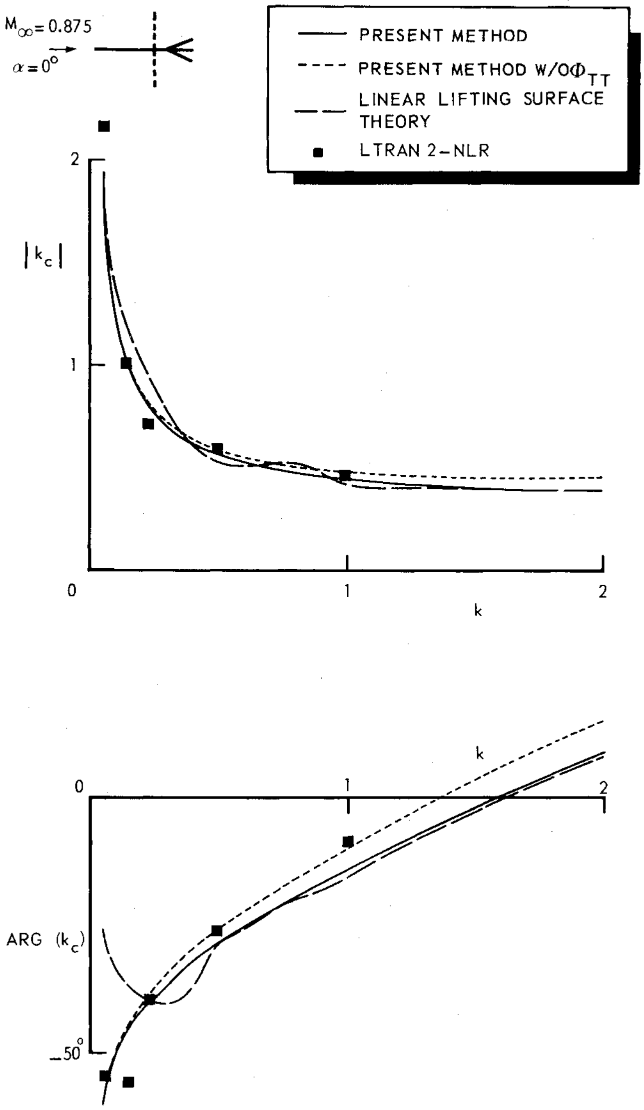


**Fig. 10 Unsteady lift coefficient on an NACA 64A006 airfoil with oscillating flap.**

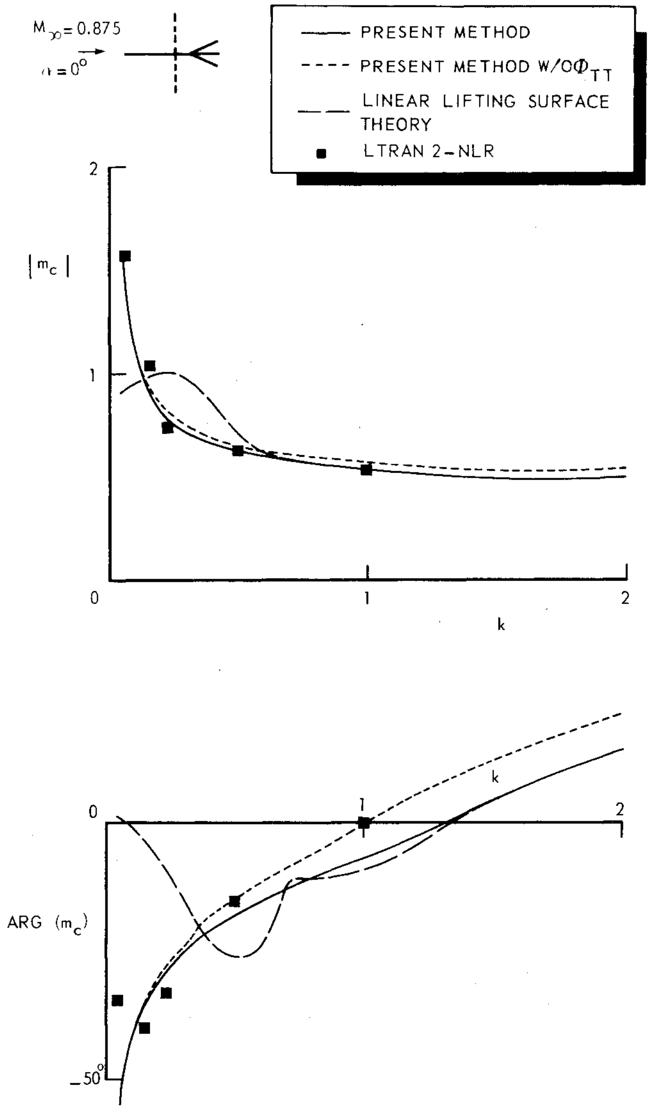


**Fig. 11 Unsteady moment coefficient on an NACA 64A006 airfoil with oscillating flap.**

the present method the results are compared with linear theory and with data obtained with the LTRAN2-NLR code for the same conditions.

For each reduced frequency $k \neq 0$ the unsteady flow has been calculated with the term $k^2 M_\infty^2 \Phi_I$ in the field equation and without this term (low frequency approximation) (see Fig. 4). This has been done for two reasons. First, a proper comparison with the LTRAN2-NLR code (low-frequency approximation) is possible and, second, the effect of the missing $\Phi_{TT}$ term in this code can be evaluated.

The steady flowfield data have been provided by the LTRAN2-NLR code (Fig. 6) and have been used to determine the supersonic Mach numbers, the shock position, and $[\![ \Phi_{0_x} ]\!]$. They are

$$M_1 = 1.12, \; M_2 = 0.875, \; X_{s_0} = 1.3 \text{ and } [\![ \Phi_{0_x} ]\!] = -0.26$$

110 panels have been used in the calculations, and a shock height of 3 chord lengths has been applied. The results have been obtained in 5% of the computer time necessary for the computation with the LTRAN2-NLR code.

### Pitching Airfoil

In Figs. 7 and 8 unsteady lift and moment coefficients are shown. As compared with the linear lifting surface results, the transonic unsteady lift and moment coefficients at reduced frequencies approaching zero exhibit a large increase in magnitude and a strong decrease in phase angle. Except in the very low reduced frequency range the present low frequency approximation (without the term $k^2 M_\infty^2 \Phi_I$) agrees satisfactorily with the LTRAN2-NLR code. This is not surprising considering the remarks made in Sec. IV. Starting at $k \approx 0.4$ the present low-frequency results and the LTRAN2-NLR results differ significantly from results of the present method.

Furthermore, at the higher frequencies there is a good agreement between the results of the present method and the linear lifting surface method. However, there remains a certain gap in magnitude.

The effect of the shock load is demonstrated in Fig. 9. At reduced frequencies approaching zero it represents the main part in the lift coefficient, while away from $k = 0$ its contribution becomes very small. This explains the difference between the transonic results and the linear lifting surface results.

### Oscillating Flap

In Figs. 10-12 unsteady lift, moment, and hinge moment coefficients are shown for the NACA 64A006 airfoil with oscillating flap. The effect of the transonic flow on $k_c$ and $m_c$ appears to be quite similar to that for the pitching airfoil. The observations made earlier concerning the comparison with

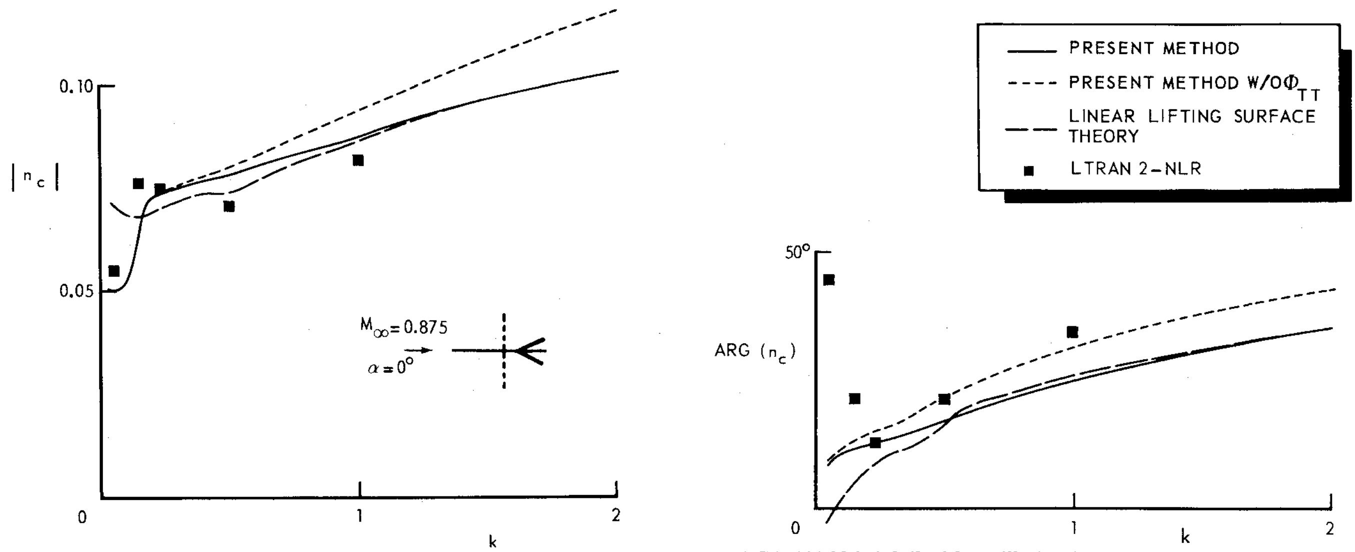


**Fig. 12 Unsteady hinge moment coefficient on an NACA 64A006 airfoil with oscillating flap.**

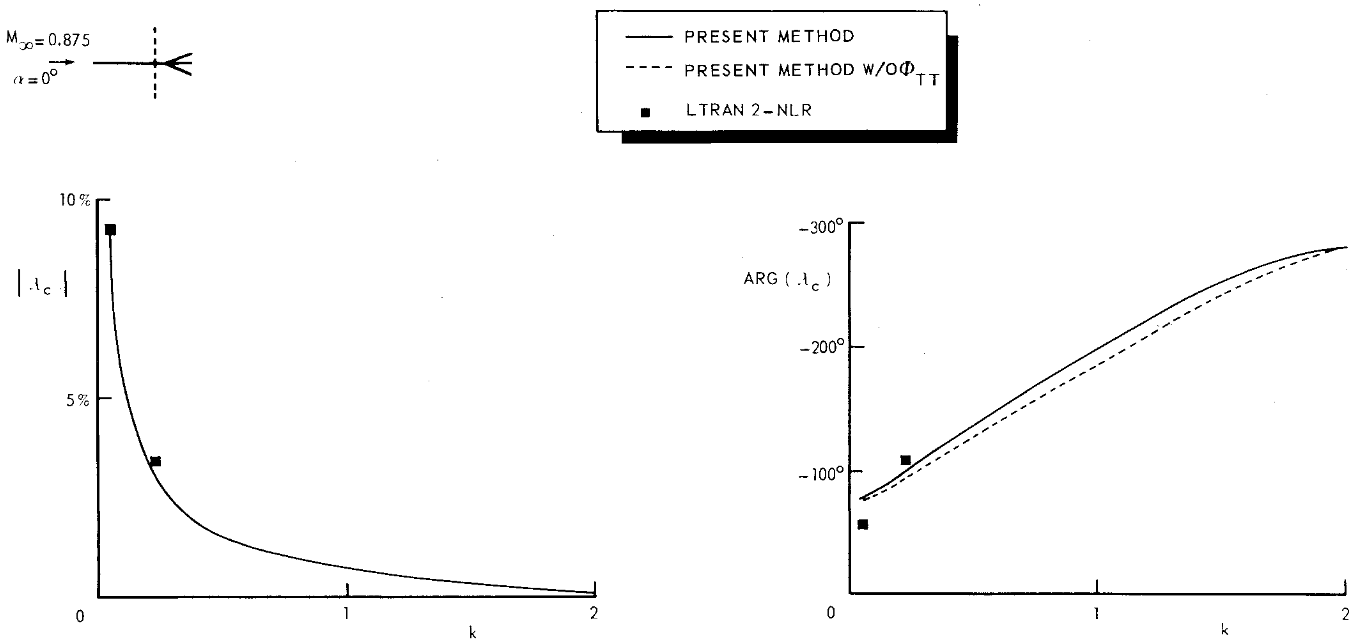


**Fig. 13 Shock displacement on an NACA 64A006 airfoil with oscillating flap, 1 deg amplitude.**

results of the LTRAN2-NLR code and linear lifting surface theory, apply also in this case.

Finally, in Fig. 13 the shock displacement, derived by assuming the solution inside the shock trajectory to be a Taylor series expansion of the steady potential, is shown [Eq. (21)]. The good agreement of the present results with the LTRAN2-NLR results justifies the assumption.

## VI. Conclusions

A relatively simple method to determine loads on thin symmetric airfoils oscillating harmonically in transonic flow has been developed by combining supersonic and subsonic panel methods in such a way that the effect of the moving shock is accounted for.

Special attention has been given to a consistent shock relation so that a better understanding of the shock load in linearized methods has been gained.

The applicability of the method has been demonstrated by comparing calculated unsteady airloads for an NACA 64A006 airfoil in pitch and with an oscillating flap with results of the LTRAN2-NLR code and linear lifting surface theory.

The calculated results have been obtained with only 5% of the computer time needed for the LTRAN2-NLR calculations.

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